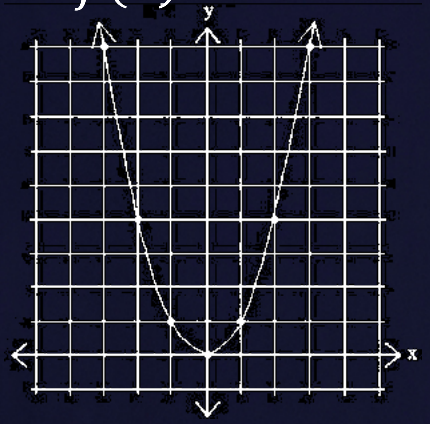
Parent Function

$f(x) = x^2$



Connecting Quadratics

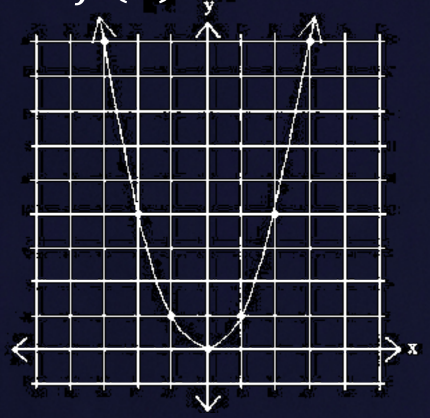
Through Completing the Square, Vertex Form, and Transformational Graphing

Jason Bragg
Christina Worley
Elizabeth Pruitt

St. Lucie
PUBLIC SCHOOLS

Parent Function

$f(x) = x^2$



Connecting Quadratics

SESSION GOALS

- Learn how multiple aspects of teaching quadratics are connected using the vertex form and transformational graphing.
- Graph quadratic functions using the vertex and symmetry.
- Connect the vertex form to transformations.
- Learn a concrete and pictorial model for completing the square.
- **Leave with a deeper understanding of the quadratics connection!**

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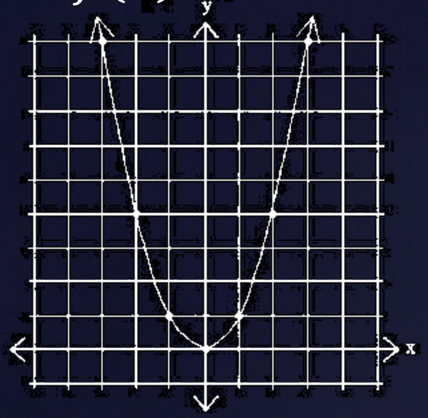


Connecting Quadratics

What is your experience with teaching Quadratics?

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Axis of Symmetry

Completing the Square

FOIL

symmetry

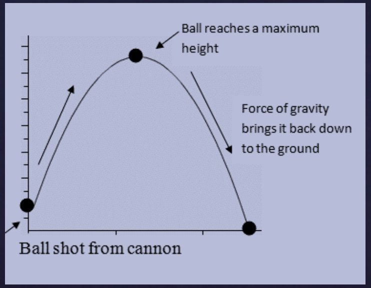
Square Root Method

vertex

Factoring

y-intercept

Gravity Problems



discriminant

parabola

x-intercepts = roots = solutions = zeros

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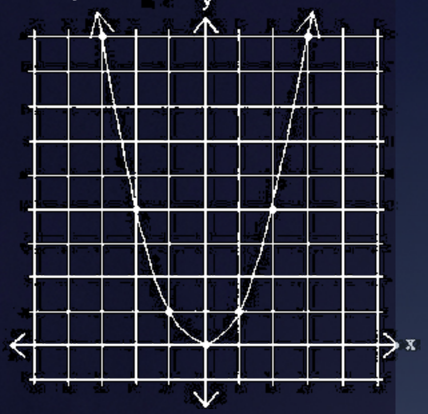
Parent Function

$f(x) = x^2$



Connecting Quadratics

How are Quadratics typically taught?

Many traditional Algebra 1 textbooks and curriculum follow this (or a similar) sequence:

1. Teach Factoring of quadratic expressions
2. Briefly look at graphing & transformations
3. Focus on "Solving Quadratic Equations"
4. Teach Completing the Square & Quadratic Formula last

But is this the optimal sequence to gain a conceptual understanding of quadratics?

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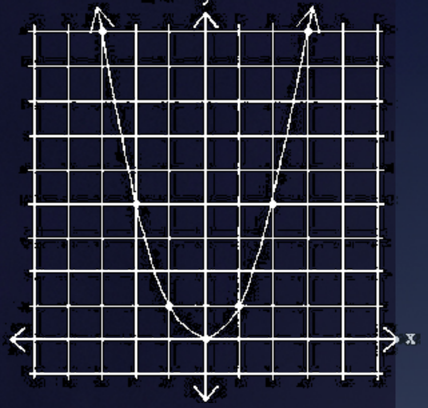
Parent Function

$f(x) = x^2$



Connecting Quadratics

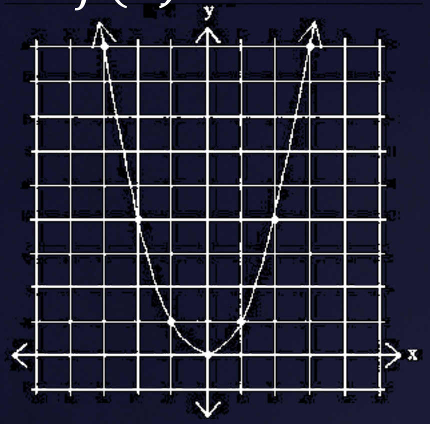
How could Quadratics be taught?

What if we aimed for coherence of these ideas?

1. Hook students with real-world connections
2. Introduce graphing & transformations
3. Connect the different forms with key properties:
   a. Vertex Form: Completing the Square to reveal the vertex, then Solving for x- and y-intercepts.
   b. Intercept Form: Factoring to reveal the x-intercepts, then using different methods to find the vertex.
   c. Standard Form: Using the Quadratic Formula to find x-intercepts, then calculating the vertex coordinates.
4. Compare multiple quadratic functions & representations

Parent Function

$f(x) = x^2$



Connecting Quadratics

Hook students with real-world connections to Quadratics

- ✓ A common situation in which we find parabolas (quadratics) are falling objects:
  - Throwing things in the air
  - Dropping things from a height
- ✓ We can construct quadratic equations in geometrical problems as well:
  - Area
  - Similar triangles
  - Right triangles (Pythagorean Theorem)
- ✓ Quadratics equations can also model pricing situations:
  - Maximizing profit
  - Minimizing expenses
- ✓ Many questions involving time, distance and speed use quadratic equations

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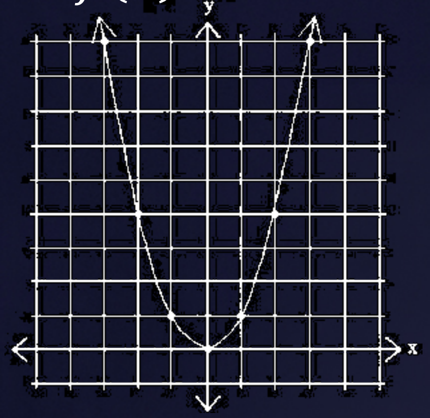
Parent Function

$f(x) = x^2$



Falling Objects – "Will it Hit the Hoop?"

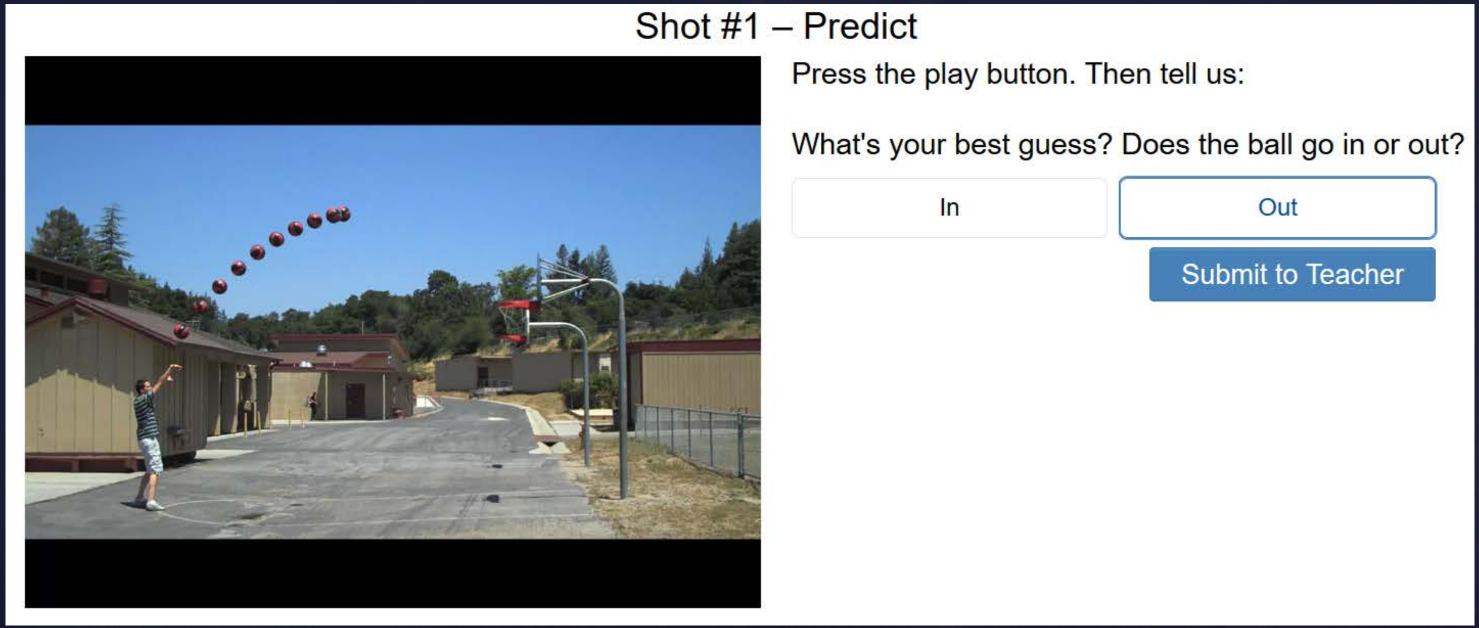
Dan Meyer created a Three-Act Task using Desmos to demonstrate this concept.

https://teacher.desmos.com/activitybuilder/custom/56e0b6af0133822106a0bed1

As a student, log in at:

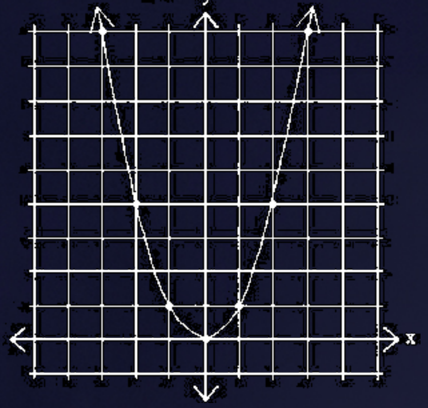
https://student.desmos.com/

(My class code: X796Z)



Parent Function

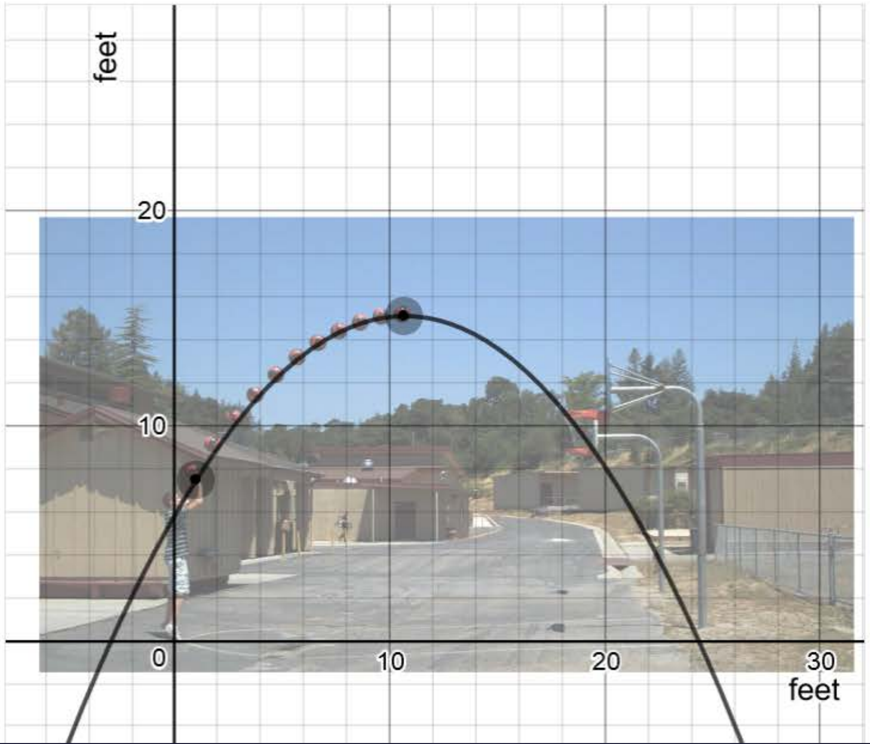
$f(x) = x^2$



Connecting Quadratics

Falling Objects – "Will it Hit the Hoop?"

Shot #1 – Analyze



Drag the black points to transform the parabola and help you decide if the ball goes in the hoop or not.

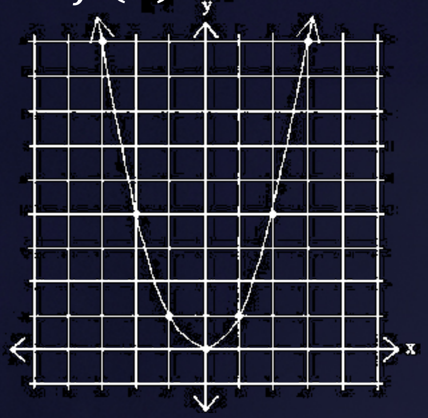
Previously you predicted the ball goes out.

In | Out

Edit your response

Parent Function

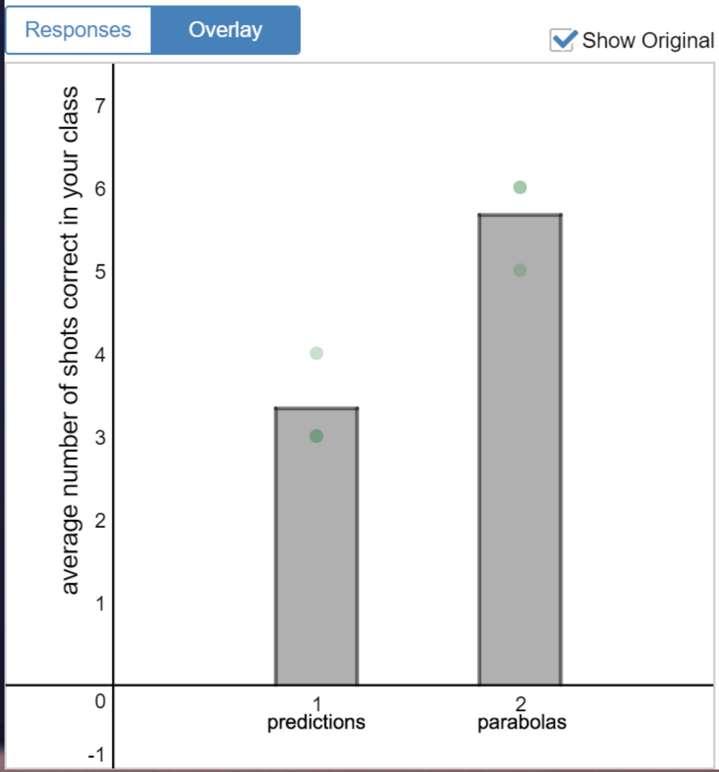
$f(x) = x^2$



Connecting Quadratics

Falling Objects – "Will it Hit the Hoop?"

Class Results



What conclusions can you draw from this graph?

(The green points represent your individual scores.)

Anthony

It looks like the parabolas are a better prediction tool.

Bethany

Parabolas are better to guess!

Charles

My predictions weren't good, but the parabolas helped

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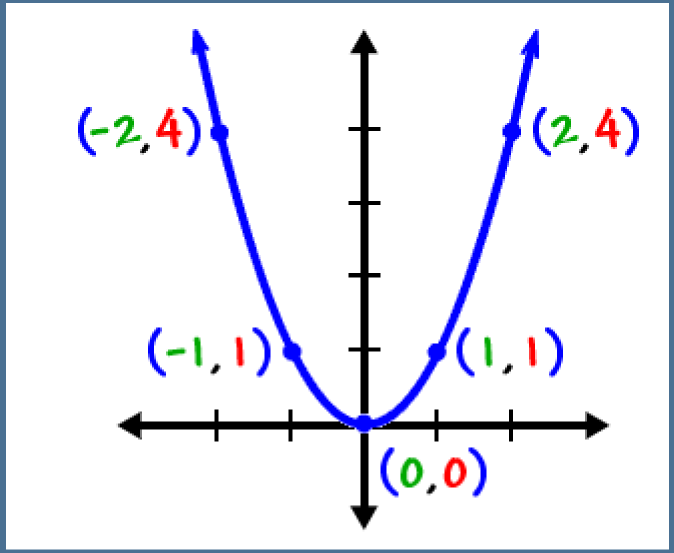
Graphing & Transformations of Quadratics

$f(x) = x^2$ is the equation of the basic parabola, aka the Parent Function

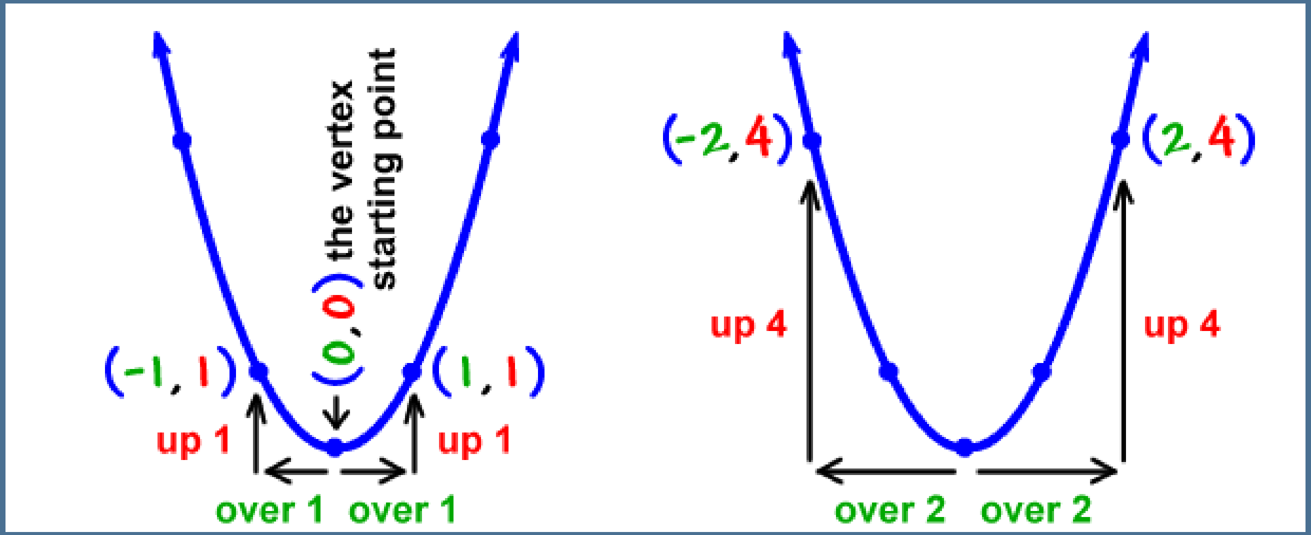
inputs		the machine		outputs	point
x	→	x^2	→	y	(x, y)
-2	→	$(-2)^2$	→	4	$(-2, 4)$
-1	→	$(-1)^2$	→	1	$(-1, 1)$
0	→	0^2	→	0	$(0, 0)$
1	→	1^2	→	1	$(1, 1)$
2	→	2^2	→	4	$(2, 2)$



Graphing & Transformations images from www.coolmath.com

Graphing & Transformations of Quadratics

Can you graph without creating a table?



What points would come next?

Graphing & Transformations of Quadratics

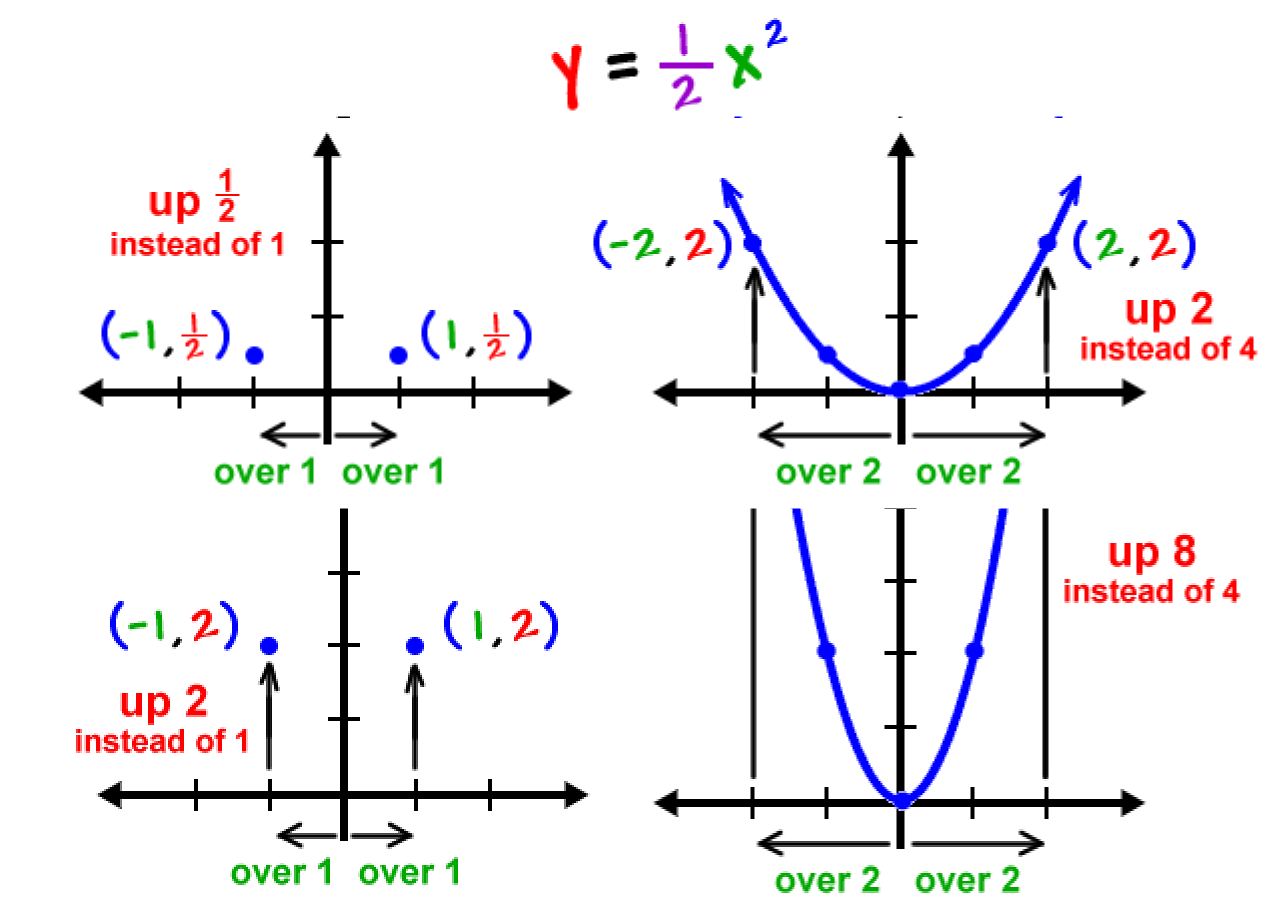
F-BF.3: Identify the effect on the graph of replacing $\boldsymbol{f(x)}$ by $\boldsymbol{f(x)+k}$, $\boldsymbol{kf(x)}$, $\boldsymbol{f(kx)}$, and $\boldsymbol{f(x+k)}$ for specific values of k (both positive and negative)…

Since using $\boldsymbol{k}$ every time can be confusing, we can use different variables for each transformation:

- $f(x)+k \quad \rightarrow \quad f(x)+\boldsymbol{k}$
- $kf(x) \quad \rightarrow \quad \boldsymbol{a}f(x)$
- $f(kx) \quad \rightarrow \quad f(\boldsymbol{b}x)$
- $f(x+k) \quad \rightarrow \quad f(x+\boldsymbol{h})$

Graphing & Transformations of Quadratics

- $f(x) + \boldsymbol{k}$
- $f(x + \boldsymbol{h})$
- $\boldsymbol{a}f(x)$
  - $-f(x)\ [\boldsymbol{a} < 0]$
  - $\boldsymbol{a}f(x)[\boldsymbol{a} > 1]$
  - $\boldsymbol{a}f(x)[0 < \boldsymbol{a} < 1]$



Graphing & Transformations images from www.coolmath.com

Graphing & Transformations of Quadratics

How do you transform the graph of a quadratic function using the parameters a, h, & k?

$$f(x) = a(x - h)^2 + k$$

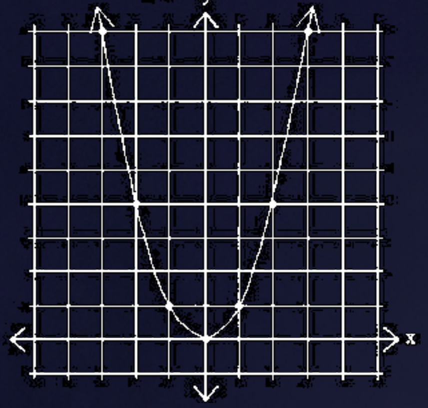
a: reflect horizontally (open up or down); stretch or compress vertically

h: translate horizontally

k: translate vertically

Parent Function

$f(x) = x^2$



Connecting Quadratics

Let's do a Quadratic sort!

- With your group, create a classifying (tree) map that quantifies different aspects of quadratics.

THINK ABOUT:

- What would you label each row?
- What are you most comfortable with as a teacher?
- What form is the most beneficial for students ?
- Do we need to address all of the forms?

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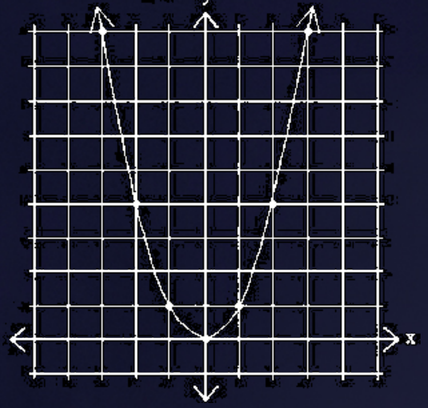
Connecting Quadratics

Parent Function

$f(x) = x^2$



Let's do a Quadratic sort!

$x^2 - 2x - 3$

Factored Form

y-intercept

Isolate variable

x-intercepts

$(x-1)^2 - 4$

Vertex Form

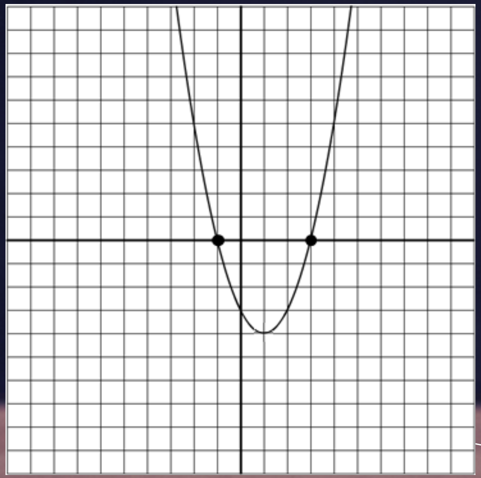
Use quadratic formula

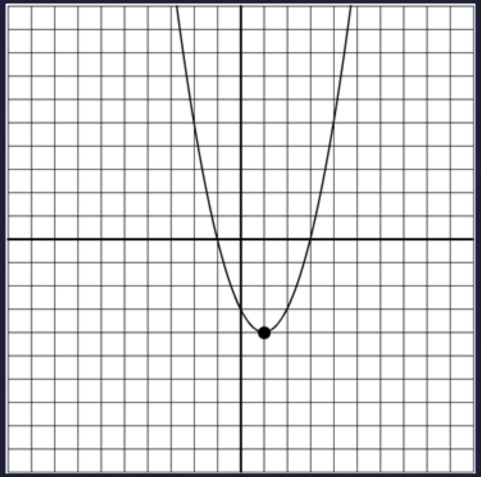
vertex

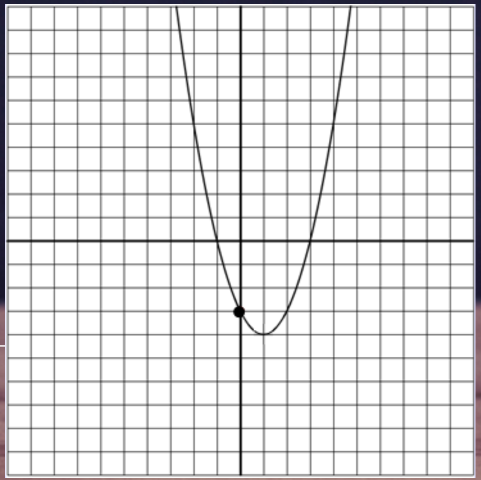
Zero Product Property

Standard Form

$(x-3)(x+1)$







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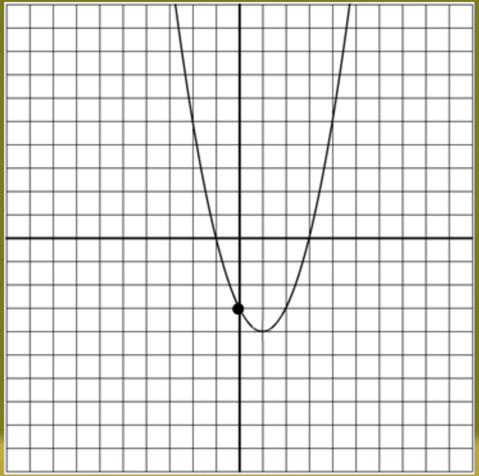
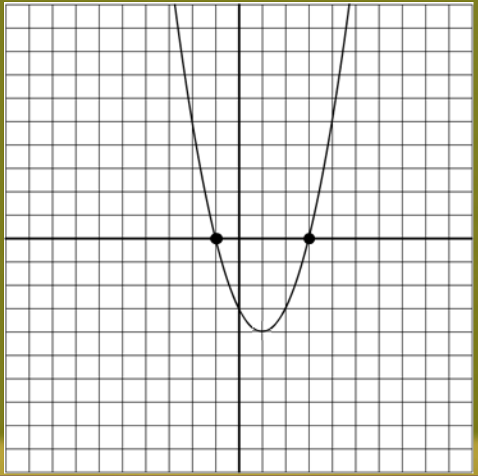
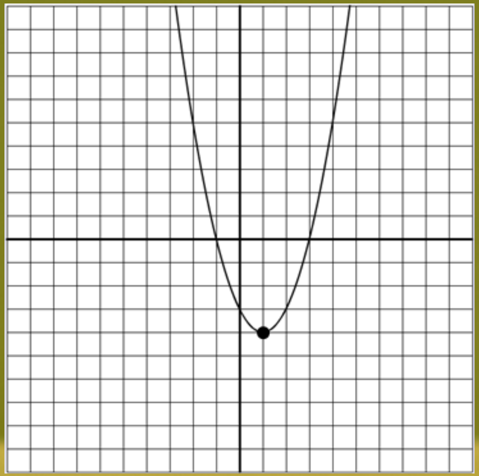
Quadratics

Type of Equation	Standard Form	Factored Form	Vertex Form
Example	$x^2 - 2x - 3$	$(x - 3)(x + 1)$	$(x - 1)^2 - 4$
Revealed Properties	y-intercept	x-intercepts	vertex
Graphs			
Finding roots	Use quadratic formula	Zero Product Property	Isolate variable

Connecting Quadratics

Parent Function

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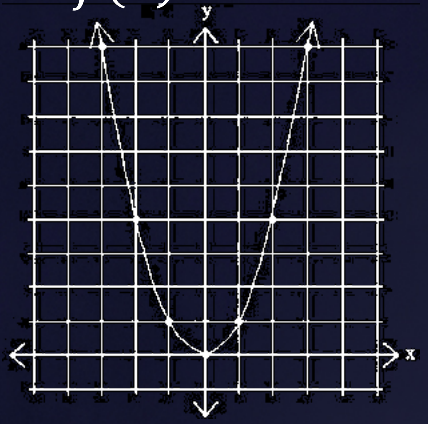

Connecting the different forms of Quadratic functions

Conversion Method: Completing the Square (Vertex ↔ Standard); Factoring (Standard → Intercept)

Quadratic Forms		Key Properties		
Name	Function	Vertex (and Axis of Symmetry)	y-intercept	x-intercept(s)
Vertex (aka Standard)				
Standard (aka General)				
Intercept (aka Factored)				

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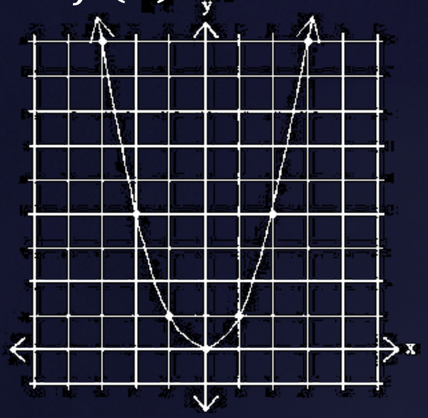
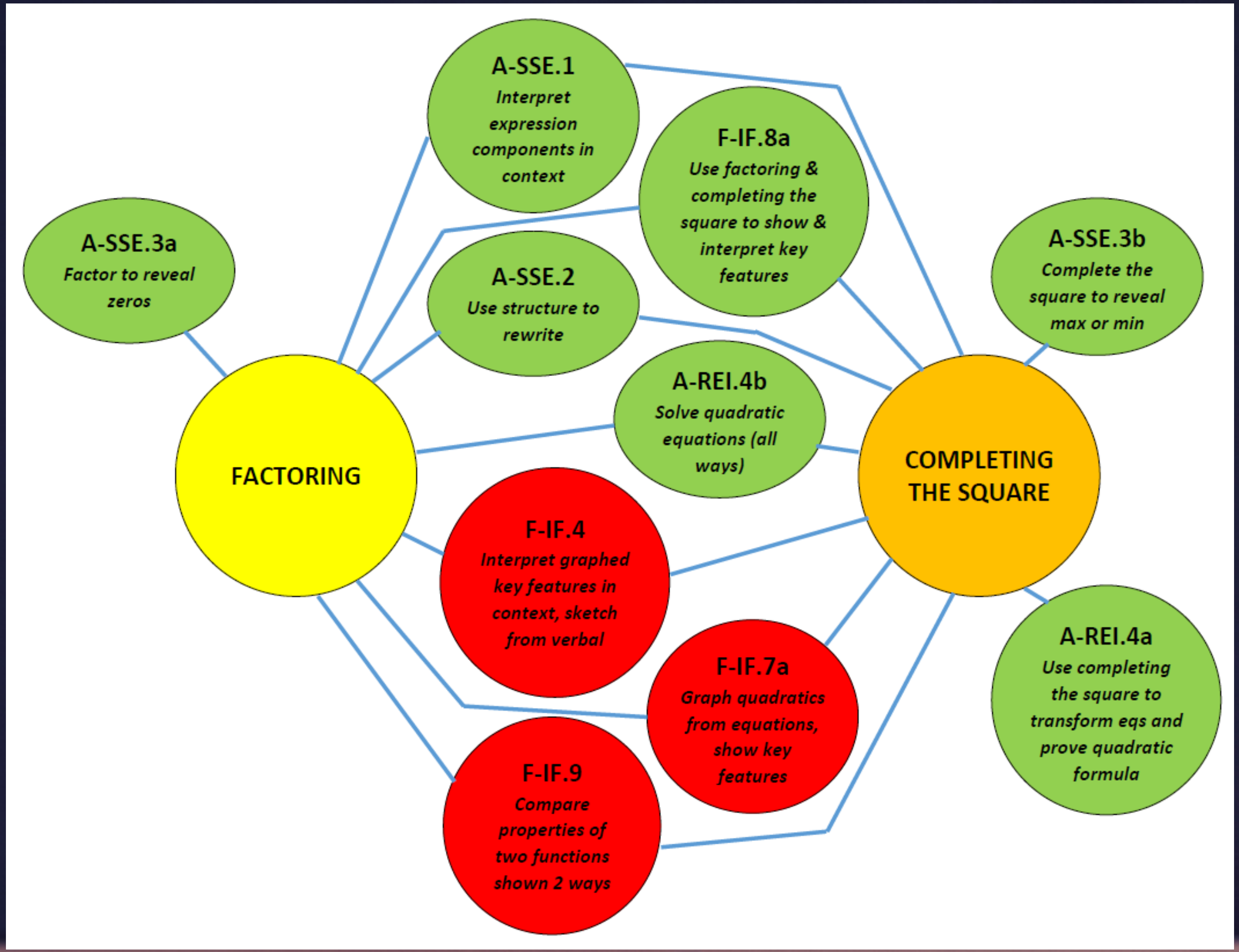
Parent Function
$f(x) = x^2$



Connecting Quadratics

The Standards concerning Quadratics: Completing the Square vs. Factoring



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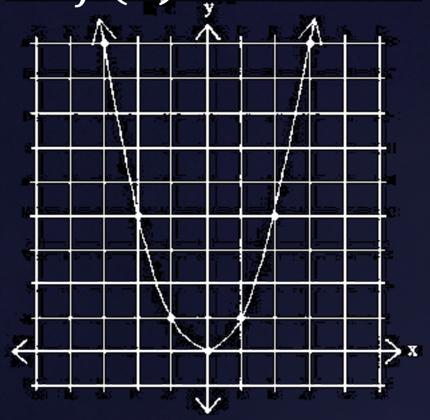


Different methods for finding equivalent Quadratic forms

- So, hopefully it's obvious by now that students need to be able to find both the vertex form and factored form of a quadratic…so how do we teach this in a way that students can grasp?

- Factoring is commonly taught using the **Area Model** (or Box Model), but Completing the Square is usually taught abstractly.

- However, using the **Area Model** to teach Completing the Square helps students grasp the conceptual understanding needed for this important method.

Completing the Square using the Area Model (to find Vertex Form)

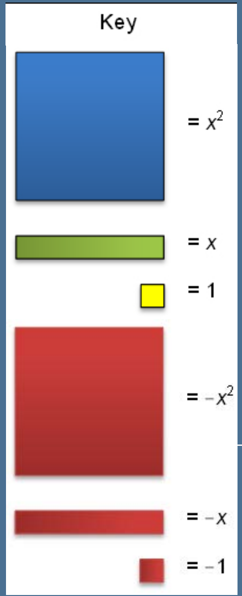
Start with a Quadratic with only a horizontal shift:

$$f(x) = (x-3)^2$$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$$f(x) = x^2 - 3x - 3x + 9$$

$$f(x) = x^2 - 6x + 9$$



Completing the Square using the Area Model (to find Vertex Form)

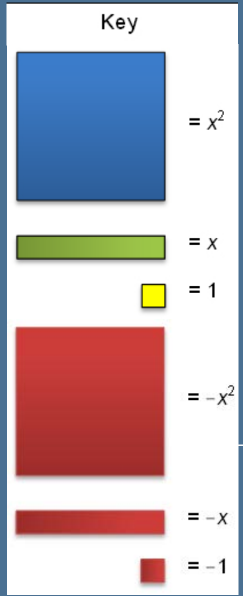
Now let's take that same Quadratic in Standard Form and change it back into Vertex Form:

$$f(x) = x^2 - 6x + 9$$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$$f(x) = (x-3)^2$$

This is where Algebra Tiles really help!



Completing the Square using the Area Model (to find Vertex Form)

$$f(x) = (x - 3)^2$$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

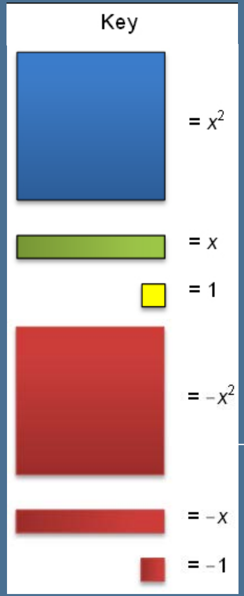
$$f(x) = x^2 - 6x + 9$$

$$f(x) = x^2 - 6x + 9$$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$$f(x) = (x - 3)^2$$

Notice that with a Quadratic with only a horizontal shift, it's quite easy to find both standard and vertex forms.



Completing the Square using the Area Model (to find Vertex Form)

But what about a Quadratic with both a horizontal and vertical shift?

$$f(x) = x^2 - 2x - 3$$

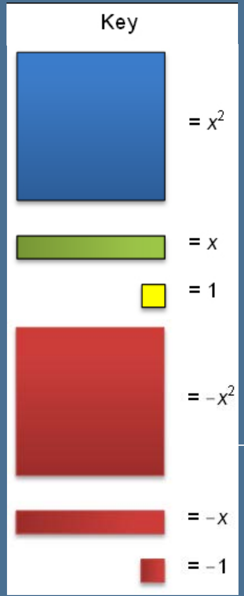
To make a "Zero Pair"

	x	-1
x	x^2	$-1x$
-1	$-1x$	1

$-1 + 4 - 3$

This is might take practice, but it's worth it!

$$f(x) = (x-1)^2 - 4$$



Completing the Square using the Area Model (to find *x*-intercepts)

$$f(x) = (x-1)^2 - 4$$

$$0 = (x-1)^2 - 4$$

$$(x-1)^2 - 4 = 0$$

$$(x-1)^2 = 4$$

$$\sqrt{(x-1)^2} = \sqrt{4}$$

$$(x-1) = \pm 2$$

$$+1 \qquad +1$$

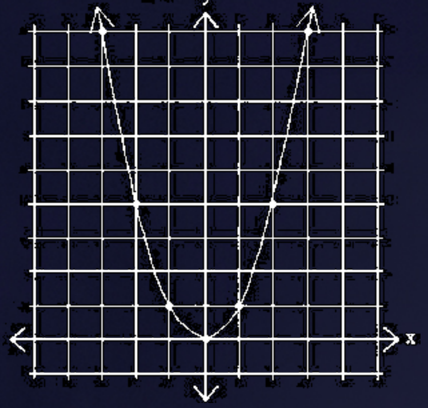
$$x = 1 \pm 2$$

$x = 1 + 2$ or $x = 1 - 2$

$x = 3$ $\qquad$ $x = -1$

Parent Function

$f(x) = x^2$



Comparing multiple Quadratic functions & representations

- Once students are fluent with converting between the different forms of Quadratics, they can compare multiple functions in multiple representations.

[Standards: **F-IF.9, A-REI.11**, and A-REI.7 (honors)]

Parent Function

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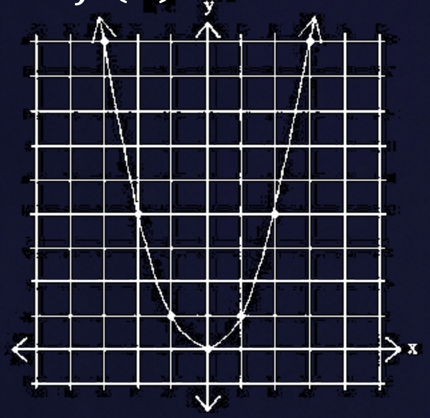

Connecting Quadratics

Did we meet our SESSION GOALS?

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- Graph quadratic functions using the vertex and symmetry.
- Connect the vertex form to transformations.
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- **Leave with a deeper understanding of the quadratics connection!**

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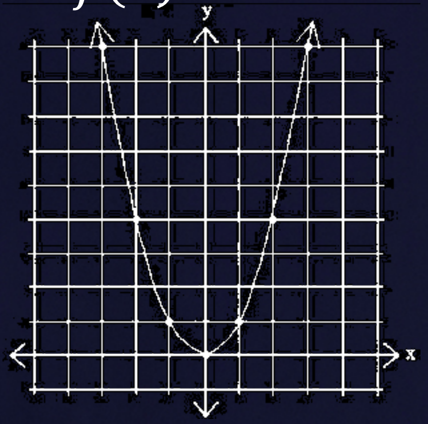

Connecting Quadratics

Resources Provided:

Scan the following QR code or use the tinyurl to access all of the resources!

https://tinyurl.com/y7tnh56x

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